The HR 550 Election Audit Methodology in U.S. Congressional Elections: Fundamental Shortcomings and Proposed Solutions

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Abstract

The purpose of this paper is to assess the accuracy and efficacy of the proposed election audit mechanism defined in HR 550, "Voter Confidence and Increased Accessibility Act of 2005," sponsored by Representative Rush Holt of New Jersey.[2] The audit requirement described in Section 5 of HR 550 calls for a mandatory hand count of 100% of the ballots in at least 2% of the precincts in each state, for each general election for federal office and, optionally, for state contests.

This paper assesses the ability of the HR 550 hand count audit scheme to reliably uncover deliberate or accidental corruption of an electronic vote tally, through two complementary methods: (1) theoretical analysis of the statistical power of the HR 550 audit protocol and (2) computer simulation of the HR 550 protocol as it would be applied in auditing elections to the United States House of Representatives.

Our key finding is that in a typical U.S. Congressional race a hand-count audit of 100% of the vote in a random 2% of the precincts *would fail about 40% of the time* to detect vote count corruption large enough to alter the outcome. This result is derived theoretically and confirmed by computer simulation.

Even in those cases where HR 550 can detect a discrepancy, there will often be only a single precinct in which corruption is detected. We question whether such a finding would be sufficient to trigger a recount, given a real world in which public perceptions have already been framed, and political pressures to accept the initial count are substantial. HR 550 offers no guidelines or criteria for a mandatory recount. Instead, the decision is left to a commission appointed by the President and inherently subject to partisan pressure.

The problem with HR 550 is not solvable simply by sampling more precincts. In order to have a 99% confidence level of detecting outcome-altering vote count corruption affecting a small number of precincts in an average U.S. Congressional District, the HR 550 protocol would need to hand-count up to *65%* of the total precincts. An alternative to HR 550 with high confidence of detection of outcome-altering vote count corruption is described, and its accuracy was also simulated; those results are presented as a potential alternative to HR 550 well worth exploring.

Since a 10% hand-count sample could be drawn in 100% of precincts on election night, the alternative has additional practical advantages: minimizing chain-of-custody concerns which are inherent to the HR 550 approach; starting the transition to universal hand-counting of elections; and placing responsibility for the integrity of the vote count in the hands of the American people, where it rightfully belongs.

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[2] http://thomas.loc.gov/cgi-bin/query/z?c109:H.R.550:

Introduction

Rationale for a mandatory independent audit of electronic vote tallies

There is a remarkable degree of consensus among computer scientists,[3] security professionals,[4] government agencies,[5] and independent analysts[6] that U.S. electronic vote tallying technology is vulnerable to unintentional programming errors[7] and to deliberate manipulation—certainly by foul-play-minded insiders at voting equipment vendors, but also by other individuals with access to voting equipment hardware or software.[8]

Federal elections determine who spends the massive federal procurement budget, who sets tax and fiscal policy for a $12 trillion economy, who drafts and enforces the United States criminal code, who sits on the United States Supreme Court, and who commands the world's largest military force in defense of our vital national interests. If there are voting system vulnerabilities, then it is a given that there would be the strongest possible motivation to attempt to exploit them to manipulate the outcome of federal elections.

Voting is a transaction extraordinarily deserving of the highest degree of protection, but it is also one that is uniquely difficult to safeguard. Electronic vote tallying systems, in particular, are inherently vulnerable to error and manipulation and it is extremely difficult to ensure that such attacks, especially when deliberate and covert, can be reliably detected or deterred.

- Voting is and must remain a *private, anonymous* transaction; voters cannot be supplied with any kind of receipt that would enable them, after the fact, to prove how they cast their vote. This common-sense measure protects voters from coercion and precludes sale of votes. But as a result we cannot employ the proven, strong, identity-based protocols that we use to secure electronic *financial* tallying.
- Voting is a uniquely important public transaction, deserving of the highest level of protection. Voting systems should be classified as *national security* systems. Successful undetected compromise of voting systems is the functional equivalent of invasion and occupation by a foreign power; the net result in either case is that the people lose control of their lives and destinies – except, of course, that an invasion is visible, and so it galvanizes resistance, while successful widespread electronic vote manipulation could well remain undetected indefinitely.

Given this constellation of threats and vulnerabilities, wherever and whenever electronic vote tallying is employed,[9] the need for a mandatory independent means of verification of the accuracy of electronic vote tallies is beyond dispute. The only question is by what means should such verification be

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[3] For instance http://www.acm.org/usacm/weblog/index.php?cat=6

[4] See the credentials of the interdisciplinary Brennan Center Task Force membership at http://brennancenter.org/programs/downloads/About%20the%20Task%20Force.pdf

[5] http://www.gao.gov/new.items/d05956.pdf

[6] See http://www.blackboxvoting.org/BBVtsxstudy.pdf , http://www.blackboxvoting.org/BBVtsxstudy-supp.pdf , and http://www.blackboxvoting.org/BBVreport.pdf

[7] Credible reports of voting equipment malfunctions are all too common; one good starting point is http://www.votersunite.org/info/messupsbyvendor.asp

[8] For example http://brennancenter.org/programs/downloads/SecurityFull7-3Reduced.pdf

[9] Hand counted paper ballots are obviously not subject to any of the risks of electronic tallying. As a case in point, see Canada: http://www.elections.ca/content.asp?section=loi&dir=leg/fel/cea&document=index&lang=e&textonly=false

performed—through electronic means, or by a hand count of a paper record? And, if so, by which one from a number of imaginable protocols?

All proposed electronic vote auditing schemes unfortunately suffer from the same basic limitation: if it provides definitive proof, after the fact, of how a voter's vote was actually cast, it violates the requirement that voting remain anonymous; if an electronic vote audit scheme does *not* provide definitive proof, after the fact, of how a voter's vote was actually cast, the fundamental issue of trust has simply been moved from one set of suspect software to another.

All electronic voting equipment ultimately translates a gesture made by a voter in the physical world (for example, touching a screen, or marking a space on an optical scan ballot) into an invisible, internal electronic tally for a candidate. No all-electronic audit mechanism can ever verify that the choice recorded internally and invisibly by the machine corresponds to the gesture originally made by the voter – regardless of any feedback that may have been provided to the voter by the machine – because, of course, machines can be programmed to lie.

For all of these reasons, the only feasible audit of electronic vote tallying is an open and observable hand count of an anonymous, tangible record of votes cast that has been verified as accurate by the voter, and stored securely and safely from the moment of casting until the time of collective verification via hand count. HR 550 is one proposed means of helping to meet the critical need of safeguarding the integrity of our electronically-tallied federal elections.[10]

Analysis

The HR 550 audit protocol

After every federal election, HR 550 prescribes a random hand count of voter-verified paper ballot records or receipts in a randomly-selected sample of 2% of the precincts in a state. The results of the hand audit are compared to the results of the electronic tally in each of the selected precincts.

This "spot audit" technique is intended to detect precincts where the electronic tally varies from an independent hand tally. If such a discrepancy is detected, either accidental or deliberate corruption of the electronic tally may have occurred. HR 550 does not specify the actions to be taken if such a discrepancy is observed, but presumably if small discrepancies occur in many precincts that consistently favor a single candidate, or large discrepancies occur in a few precincts, there is a possibility that the outcome of an election may have been changed. Though no standards are given, HR 550 provides that if the presidentially-appointed Election Assistance Commission (EAC) finds that these spot audits "*show cause for concern* about the accuracy of the results of an election" (emphasis ours), the EAC may choose to pursue further investigation.[11] Again at the discretion of the EAC, the hand audit could be expanded, possibly to the point where a full hand recount of the paper ballot records is performed. Depending on the applicable state law, the outcome of the hand audit, and political considerations, the legitimacy of a suspect election could be challenged.

Advocates of HR 550 have asserted that the ability to detect electronic vote count corruption and to challenge the outcome of disputed elections could help restore public confidence in the electoral

[10] And, optionally, state elections as well. The focus of this paper is on US Congressional races.
[11] HR 550, Sec. 5 (d).

process—or possibly trigger broader reforms of the election process if many such problems are uncovered over time.

But before we decide whether to adopt HR 550, we need at a minimum to understand whether, and under what circumstances, its audit protocol can reliably detect vote count corruption.

At first glance it seems plausible that selecting a random sample of 2% of the precincts in a state might reliably uncover *some* evidence of problems with the official electronic vote tally, but a means of quantifying the *actual* likelihood of detecting vote count corruption is needed.

Theoretical limitations of the HR 550 election audit protocol

The theoretical accuracy, or "hit-rate," of any election spot-audit scheme is a function of three quantities: the total number of precincts in a jurisdiction; the number of precincts in the random sample; and the number of precincts where the electronic tally has, through accident or design, been corrupted.

The theoretical accuracy of a spot-audit can be expressed as the probability that a sample T of a total number of N precincts will detect at least one of F precincts with a corrupted vote count. That probability is calculated as follows:[12]

Probability of detection (P) = $1 - ((N - F)!\,(N - T)!) \,/\, (N!\,(N - F - T)!)$ if $F+T \bullet N$;

and, as a practical approximation, if $F+T << N$ then $(P) = 1 - (1 - F/N)^T$

A spreadsheet that calculates the probability of detection of accidental or deliberate vote count corruption given a total number of precincts, the total number of spot-audit precincts, and the total number of corrupt precincts is available for download[13].

This formula can be used to estimate the ability of HR 550 to identify potential problems with the integrity of U.S. Congressional races. As data from the EAC reveals, the average U.S. Congressional district contains about *440 precincts* (see Appendix A).

Typically, an election is not stolen by small actions spread evenly across a district. There are precincts where a party's control of the local machinery makes large-scale fraud a temptation and other precincts, with close bipartisan monitoring, where the probability of detection is sufficient to deter fraud. As an example, we imagine a scheme confined to 10% of precincts in a Congressional district, where a large proportion of votes cast in those precincts are fraudulently shifted, but 90% of precincts are counted honestly.

Substituting N = 440, F = 44, and T = 8 into the formula above yields a probability of detection of 57.3% (or 61.3% if, to satisfy the 2% sampling criteria, we round up and spot-check 9 precincts instead of 8). That means we would fail to detect *any* discrepancy in between 39.7% and 42.7% of elections with fraud or error in one out of ten precincts.

[12] A lucid derivation of this formula is at http://www.votehere.net/papers/ElectionConfidence.pdf ; note that we do not endorse the proposed VoteHere election audit solution, but simply note that the mathematics underlying the election confidence formula are quite clearly explained.

[13] www.electiondefensealliance.org/downloads/election_audit_probability.xls

In other words, HR550 would *completely fail to detect* deliberate fraud or accidental mistabulation affecting one out of ten precincts in an average U.S. Congressional District race about *40%* of the time. This degree of vote count corruption could easily be large enough to change the outcome.

This finding is a direct consequence of the relatively small number of precincts in an average U.S. Congressional district. The success of a precinct sampling system depends on the *absolute number* of sampled precincts. Although 2% may be a valid sample in other circumstances, in the case of a Congressional district this translates to only 8 or 9 precincts sampled, which is an inadequate safeguard.

We created an election simulation computer program[14] to check the accuracy of the theoretical prediction. The election simulation was programmed to reproduce typical U.S. Congressional races with approximately 100,000 – 300,000 simulated voters, in venues comparable to the average U.S. Congressional District size of 440 precincts, plus or minus 140 precincts.

The results from 10,000 simulated U.S. Congressional races along with the corresponding values predicted in the spot-audit accuracy formula are shown below.

Parameter	Theoretical value from formula	Observed value in simulation
Average U.S. Congressional District number of precincts	440	439.7925 average over 10,000 elections
HR 550 number of precincts to audit (@ 2%)	8 (truncate 8.8) or 9 (round up 8.8)	8.78 average over 10,000 elections
Percentage of elections where HR 550 audit protocol *totally fails to detect* simulated fraud	42.7% (audit 8 precincts) or 39.7% (audit 9 precincts)	40.68% i.e., significant fraud in 4,068 out of 10,000 elections

Clearly the simulation results confirm the accuracy of the theoretical formula.

In addition to the very large number of corrupted US House elections that the HR 550 audit protocol would completely fail to detect, there are additional concerns with its accuracy and practical efficacy.

HR 550 audits can seriously understate the incidence of vote count corruption
Even in those cases where the HR 550 protocol detects *at least one* precinct with a vote count discrepancy, in many instances the magnitude of the problem it reveals is significantly underestimated.

Our simulation created an average shift of 5.1% of the total vote from one candidate to the other. That average degree of simulated fraud would easily be sufficient to change the outcome of a competitive election—turning, for example, a candidate's clear 54% to 46% victory to a close 49% to 51% loss.

Even so, in about 45% of those simulated corrupted elections, the HR 550 audit detected a discrepancy of less than 1%. The problem with significant underestimation of vote count corruption is, of course, that apparently small or "minor" problems can be readily dismissed. This is especially troubling

[14] The details of the election simulation program design are described in Appendix B below, in sufficient detail to enable a computer programmer to produce an equivalent program to verify the results described here.

because, as noted previously, HR 550 establishes no standards whatsoever regarding what size discrepancies are significant enough to be actionable, leaving the matter entirely to the unguided discretion of the EAC.[15] Nor does HR 550 provide firm guidelines for the expansion of the audit when a "significant" discrepancy *is* found. As a result, even when HR 550 finds evidence of significant fraud or mistabulation, as a practical matter the corrupt results often would remain unchallenged.

HR 550 audits cannot be used to estimate the total incidence of vote count corruption

The key challenge in auditing election results is to know with high certainty whether the official electronic tally was altered and, if so, by how much.

As we have already seen, in U.S Congressional races HR 550 can give only a partial answer to the first question—was the electronic tally altered?—by sometimes detecting that the vote count in one or a few precincts was inaccurate.

But the answer to the second question–by how *much* was the tally altered?–is effectively *impossible* to determine in a HR 550 audit. A random sample of 2% of the precincts in a Congressional race, as a result of clustering effects,[16] is simply insufficient to accurately extrapolate the total vote across the entire set of precincts.

This fundamental inaccuracy is illustrated by the chart in Figure 1.

[15] Since experience has shown that interpretation of voter intent means that hand and machine counts for a precinct often differ slightly as a matter of course, thus making a "zero-discrepancy" standard impractical, it appears that *any* standard added to HR 550 would in fact be arbitrary. HR 550, in short, offers no basis for separating fraud from noise, and such a basis is difficult if not impossible to provide in such a spot-audit scheme.

[16] For an attempt to quantify the effect of clustering, or drawing samples from a few "convenient" locations rather than proportionately from the whole, see Merkle, D. and Edelman, M. "A Review of the 1996 Voter News Service Exit Polls from a Total Survey Error Perspective," in *Election Polls, the News Media and Democracy*, ed. P.J. Lavrakas, M.W. Traugott, New York: Chatham House, pp. 68 – 72.

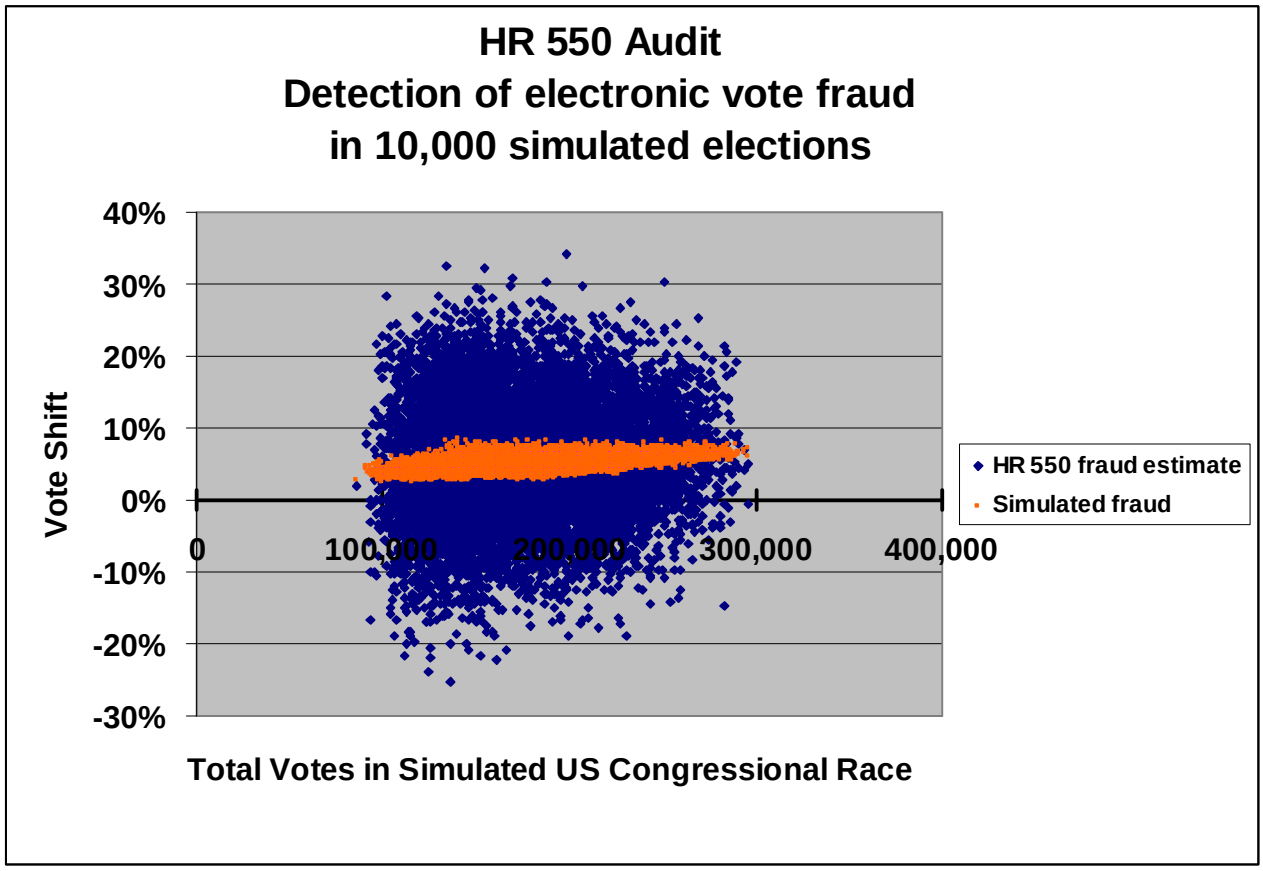

Figure 1: HR 550 candidate totals provide no basis for estimating the true size of a district-wide vote count discrepancy

As can be readily seen, the vote tally for a candidate in the HR 550 sample precincts (blue dots) can grossly underestimate or overestimate a candidate's district-wide vote. In comparison, the orange dots represent the variance due to vote fraud: the difference between the paper ballot tally and the electronic tally of the candidate in the election as a whole. The HR 550 audit's estimate of a candidate's uncorrupted vote was outside the actual range of simulated vote count corruption about *76%* of the time.

Due to inherent clustering problems, you simply can't use the results of an HR 550 audit to estimate the *degree* of vote count corruption in a Congressional race. At best HR 550 can only hint at the *presence* of vote count corruption by detecting one or more precinct-level discrepancies.

American political culture hardly encourages candidates to challenge election results in the absence of compelling, objective evidence of a widespread problem. If there is no way for a candidate to quickly know not only that the count was off in a few precincts, but also *how much total* vote count corruption occurred, candidates will simply not possess the information necessary, as a practical matter, to challenge the outcome of an election. Any further pursuit of the matter would be left to the standardless discretion of the politically-appointed EAC. Such a protocol does little to shore up voter confidence in the accuracy and honesty of the vote-counting process.

99% confidence of detecting vote count corruption is not feasible in the HR 550 protocol

One might well ask whether it would be possible to sample more precincts, and so extend the HR 550 sample to gain such a high degree of confidence of detecting both the presence and the total amount of vote count corruption in Congressional races that candidates could reasonably insist on a full recount. A 99% confidence level of detection of significant fraud, for example, would clearly enable candidates to mount a credible challenge.

In order to have 99% confidence, however, that an HR 550 audit can detect vote count corruption affecting 10% of the total precincts in an average U.S. Congressional race, the theoretical model predicts that a random sample of *9.5%* of all precincts would need to be hand-counted.[17] Vote count corruption in 5% of the total precincts would be more than enough to swing a close race. To detect corruption of 5% of the precincts in a U.S. Congressional race with 99% certainty, the HR 550 audit would need to hand-count *19%* of all precincts.[18] To achieve 99% certainty of detection of vote count corruption in 1% of total precincts (still sufficient to swing a close race) the HR 550 audit would require hand-counting *65% of all precincts*![19]

At that hand-count sampling rate, of course, there is hardly a rationale for continued use of electronic vote tallying technology.[20] Until hand-counted paper balloting is universal, do we truly have to accept living in a country where we will never know for sure whether the victor in *any* close Congressional race was really the choice of that district's voters?

99% confidence of detecting 1% vote count corruption *is* achievable - without HR 550

There is a simple alternative audit protocol that *can* immediately achieve 99% confidence of detection of manipulation of even 1% of the total vote: *hand count 10% of the paper ballot records in a U.S. Congressional District race in 100% of the precincts.* We refer to this as a "Universal Precinct-based Sampling," or UPS.

Besides its extraordinary accuracy, UPS has the additional advantage of being simple enough to be done in-precinct, on election night, by public volunteers representing all concerned political parties. If done in-precinct,[21] this alternative avoids the difficult issue of guaranteeing integrity of the chain of custody of the paper ballot records in all 180,000 U.S. precincts as required by the HR 550 protocol.

The practical details of UPS are beyond the scope of this paper, and will be presented in full elsewhere.[22] However, we can easily examine the comparative accuracy of a Universal Precinct-based 10% hand count sample in 100% of precincts, by testing it against the same ten thousand simulated U.S. Congressional-sized races and the same simulated fraud scenarios that we presented to the HR 550 audit.

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[17] N=440, F =44, T=42, probability of detection = 99.0292%

[18] N=440, F =22, T=84, probability of detection = 99.1947%

[19] N=440, F=4, T=286, probability of detection = 99.0449%

[20] Fully hand-counted paper ballots would clearly be a much simpler, more secure, and overall more appealing alternative. We endorse 100% hand-counting as superior to any audit protocol – any partial audit is only a transitional step to that goal. Accommodation of voters with visual or mobility impairments as mandated by HAVA, does not, of course, require any use of computerized vote tallying technology. See http://www.vote-pad.us/index.asp

[21] Done at the precinct level, this protocol would sample from all ballots, including early and absentee, as well as those cast in-precinct on Election Day.

[22] This paper, entitled "Universal Precinct-based Hand Count Sampling Protocols and Procedures," is to be published shortly at www.ElectionDefenseAlliance.org.

In comparison with the HR 550 results shown above in Figure 1, the UPS protocol not only detects *100%* of the cases of simulated vote count corruption, it also correctly estimates the *size* of the simulated vote count discrepancy.

In a side-by-side comparison against the same set of simulated fraudulent elections, UPS dramatically outperformed HR 550. The UPS estimate of vote count corruption was accurate to within ± *1%* of the actual value with a certainty better than 99.7%.

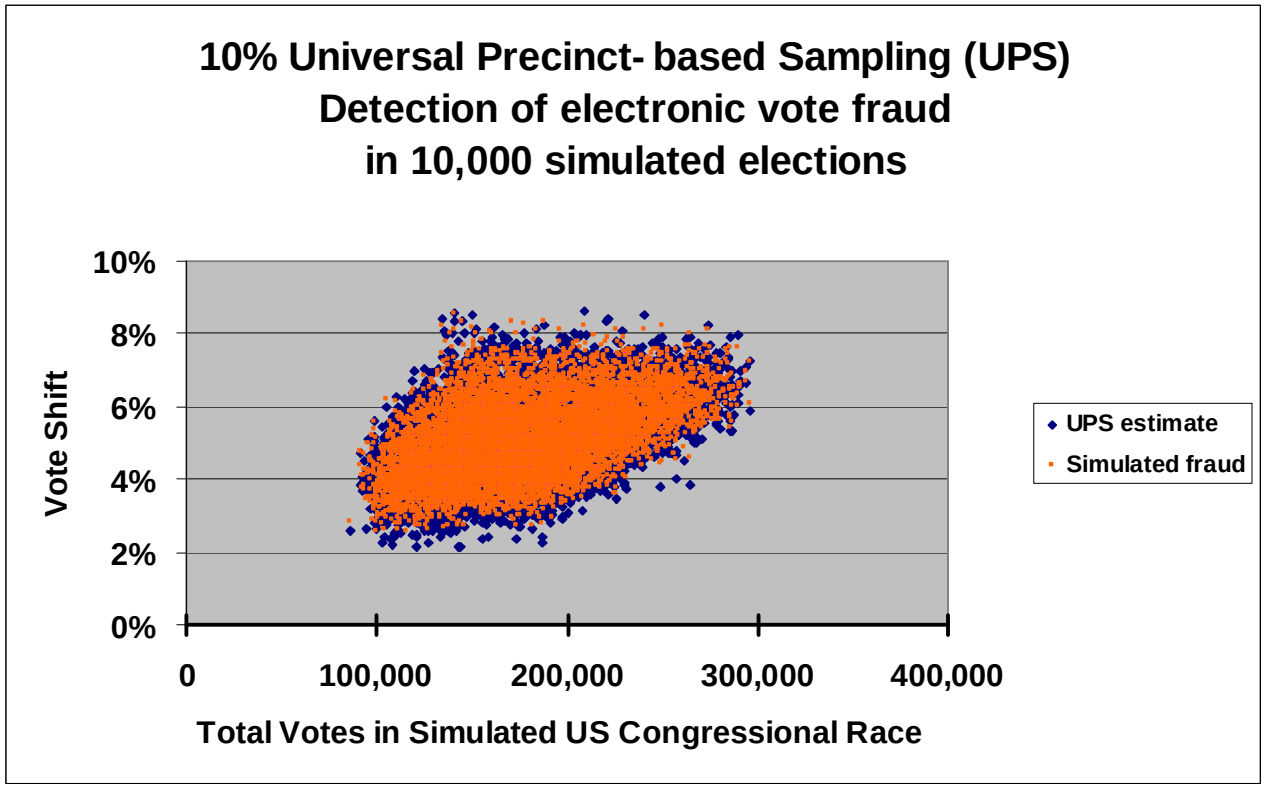

Figure 2: Compare to Figure 1; the same simulated vote fraud data was presented to both protocols. In comparison with HR 550, hand-counting 10% of paper records in all precincts is *extraordinarily* accurate in detecting and estimating the true magnitude of fraud.

Parameter	HR 550 audit	UPS Protocol
US Congressional District average number of precincts	439.7925 average over 10,000 elections	439.7925 average over 10,000 elections
Minimum and maximum simulated fraudulent vote shift	2.5% to 8.5% vote shift	2.5% to 8.5% vote shift
Number/percentage of 10,000 simulated fraudulent elections detected by audit	59.32% detection rate 4,068 cases of fraud not detected	100% detection rate 0 cases of fraud not detected
Number of elections where total vote shift was estimated to ± 1% accuracy	842 out of 10,000	9,972 out of 10,000

Conclusion

The HR 550 audit protocol is, in theory and practice, unable to guarantee the integrity of U.S. Congressional elections.[23]

Pending full transition to hand-counted paper ballots, we should adopt a statistically robust, pragmatic hand-count sampling alternative – to be conducted in precinct, on election night, by and for the American people.

[23] HR 550's performance in statewide contests varies with the size of the state but is in all cases inferior to that of UPS in detecting both the presence and the magnitude of vote mistabulation.

APPENDIX A. Statistical profile of U.S. Congressional Districts

The following data are extracted from Election Assistance Commission data published at http://www.eac.gov/election_survey_2004/chapter_table/Chapter13_Polling_Places.htm

Name		Electoral Votes	Precincts per Congressional District	Total Number of Precincts
Alabama		9	315.7143	2210
Alaska		3	436	436
Arizona		10	263.75	2110
Arkansas		6	673.25	2693
California		55	412.3962	21857
Colorado		9	481.4286	3370
Connecticut	(1)	7	153.8	769
Delaware		3	437	437
District of Columbia		3	142	142
Florida		27	275.68	6892
Georgia		15	243.3077	3163
Hawaii		4	176.5	353
Idaho		4	474.5	949
Illinois		21	617.7895	11738
Indiana		11	619	5571
Iowa		7	393.2	1966
Kansas		6	970.5	3882
Kentucky		8	580.3333	3482
Louisiana		9	589.1429	4124
Maine		4	300.5	601
Maryland		10	222.375	1779
Massachusetts		12	217.7	2177
Michigan		17	349	5235
Minnesota		10	513.5	4108
Mississippi		6	426.75	1707
Missouri		11	606.8889	5462
Montana		3	856	856
Nebraska		5	556	1668
Nevada		5	528.3333	1585
New Hampshire	(2)	4	150	300
New Jersey		15	483.3077	6283
New Mexico		5	228	684
New York		31	522.5172	15153
North Carolina		15	211.4615	2749
North Dakota		3	607	607
Ohio		20	631.4444	11366
Oklahoma		7	430.4	2152
Oregon		7	289.6	1448
Pennsylvania	(3)	21	494.3158	9392
Rhode Island		4	288.5	577
South Carolina		8	361.3333	2168
South Dakota		3	827	827
Tennessee		11	254.1111	2287
Texas		34	267.3125	8554
Utah		5	626.6667	1880
Vermont		3	277	277
Virginia		13	208.5455	2294
Washington		11	740.4444	6664
West Virginia		5	659	1977
Wisconsin		10	445.375	3563

Name	Electoral Votes	Precincts per Congressional District	Total Number of Precincts
Wyoming	3	483	483
Total			174252
Maximum		970.5	21857
Average		437.6211	3485
Minimum		142	30

Note: All data from EAC as cited except for Connecticut, New Hampshire and Pennsylvania. Precincts per Congressional District is derived.

(1) – Data from http://www.verifiedvoting.org/downloads/CT_Cost-analysis-model.pdf
(2) – Data from http://www.invisibleida.com/New_Hampshire.htm
(3) – Data from http://jsg.legis.state.pa.us/ELECTION.PDF

APPENDIX B. Technical description of election simulation program

Election Simulation Technical Overview

Although statistics alone can describe in detail the expected accuracy of the sampling techniques that can be applied to election verification, whenever ideal, mathematically-continuous techniques are applied to the discrete events of the real world, there is a potential for inaccuracy.

An election simulation allows us to model the essential features of real world elections, accounting for such factors as variable precinct size, differing precinct partisanship, and large and small electorates. Computers can easily conduct thousands of simulated elections in a short period of time, far more than could ever be measured or studied in the field. And simulated voters have no expectation of privacy or anonymity.

We not only can collect and count simulated ballots precisely as cast, but also can simulate a variety of different methods for electronic vote manipulation–and then test the ability of *any* audit or verification protocol to reliably detect the both the scope and magnitude of the simulated fraud.

Basic election simulation algorithm

Contests with two candidates are sufficient to investigate the accuracy of election auditing protocols. This election simulation consists of a series of thousands of contests between Candidate A and Candidate B.

Parameters within the program allow a larger or smaller number of precincts to be created from election to election. Each simulated election is conducted in between 1 and 10,000 simulated precincts. Each precinct ranges in size between 1 and 1,000 voters, so elections with as many as ten million votes can be simulated. By altering the parameters that control the allowable number of precincts, it is easily possible to simulate US Congressional elections with 200,000 to 250,000 total votes; medium-sized statewide contests with 1,000,000 to 4,000,000 votes; or even elections in large states with 10,000,000 votes.

Variations in partisanship are also simulated. Each precinct's partisanship is randomly chosen, but the range of average, minimum and maximum partisanship allowable across all precincts can be varied between elections. This means that by altering program parameters it is possible to simulate elections where Candidate A is favored, Candidate B is favored, or where the two candidates are closely matched.

The basic election simulation algorithm is as follows:

```
Set up a new election:
Initialize working memory
Obtain the parameters that control overall election size and partisan mix
Determine the number of precincts (1 to 10,000)
For each simulated precinct
      Determine the size of the precinct (1 to 1,000)
      Determine the Candidate A partisanship index of the precinct (a number
      between 0 and 1)

Conduct the election:
      For each simulated precinct
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            For each simulated voter in the precinct
                  Determine the partisanship of the voter (a number between
                     0 and 1)
                  If the voter's partisanship is less than or equal to the
                     precinct's Candidate A partisanship index,
                        cast a vote for Candidate A
                  else
                        cast a vote for Candidate B
```

The simulated votes are stored in a two-dimensional array (precinct by precinct size). After all votes are cast in all simulated precincts, the results of the election are tallied and displayed.

Description of the HR 550 audit algorithm

The algorithm simulating the HR 550 protocol – in this case, a sample of 100% percent of the votes in a random 2% of the total precincts was straightforward. A very simple model of clustered vote corruption was simulated where sufficient votes are shifted in 10% of the total precincts to transfer between 3% and 8% of the total vote from one candidate to another.

```
Simulate vote count corruption:
      Set the number of corrupt precincts to 10% of the total precinct count
      For each precinct to be corrupted, select a unique random precinct number
      For each precinct in the random list of precincts to be corrupted
            Randomly change 30 – 80% of the precinct's cast votes from Candidate A
            to B to cause a jurisdiction-wide shift of 3% – 8% of the total vote
            Save the corrupted votes for the precinct as the electronic tally for
            that precinct
      For all other precincts
            Copy the original precinct's cast votes as-is and save as the
            (uncorrupted) electronic tally for that precinct

Set up a new HR 550 audit:
      Initialize the working memory that will contain the audit totals
Conduct the simulated HR550 audit:
      Calculate the number of precincts to audit as 2% of the total precinct count
      For each precinct to be audited, select a unique random precinct number
      For each precinct in the random list of simulated election to be audited
            For each ballot cast in the precinct
                  If that ballot is marked for Candidate A
                        Increment Candidate A's audit total
                  If that ballot is marked for Candidate B
                        Increment Candidate B's audit total
            After tallying the simulated paper ballots in an audited precinct
                  If the precinct simulated paper audit tally is different than
                  the precinct simulated electronic tally
                        Increment the tally of precincts having a vote count
                        discrepancy
                        Add the precinct vote count discrepancy to the total vote
                        count discrepancy

Report the HR 550 audit results:
      When all precincts have been sampled, print out the audit totals for
            Candidate A and B
      Compare the audit totals for Candidate A and B with the actual totals for
            Candidate A and B to assess the accuracy of the audit sample
```

Description of the Universal Precinct-based Sampling algorithm
The alternative UPS election verification protocol is simple. Given a paper optical scan ballot or voter-verified paper trail,[24] select and hand-count a random sample of paper vote records from *every precinct*, and then compare those aggregated totals to the official electronic tally.

Basic statistics tells us that if the random sample sizes are adjusted properly, it will be possible to detect even a small amount of deliberate fraud or inadvertent error that causes the official electronic tally to deviate from the paper audit trail, with a very high degree of confidence.

One goal was to measure the size of the hand-count sample that would allow us to be 99% certain that the result of hand-counting a random sample of the paper trail was within 1% of the actual results. Proving we can achieve this level of accuracy would help ensure that we would not undertake the effort of a full manual recount of all ballots unless there was what amounts to 100-to-1 odds that there was a serious problem with the official electronic count.

The algorithm implementing a random selection of a percentage of the votes in every precinct is straightforward.

```
Set up a new hand-count sampling:
      Initialize the working memory that will contain the hand-count sampling
      totals
Conduct the hand-count sampling:
      For each precinct in the simulated election
            Calculate the number of ballots to be sampled in the precinct(equal to
            the sampling percentage times the number of cast ballots)
            For each ballot to be sampled
                  Calculate x = a unique random number between 1 and the number of
                  cast ballots in the precinct
                  Examine the x^th ballot in the precinct
                        If the x^th ballot is marked for Candidate A
                              Increment Candidate A's audit total
                        If the x^th ballot is marked for Candidate B
                              Increment Candidate B's audit total
Report the hand-count sampling results:
      When all precincts have been sampled, print out the hand-count sampling
            totals for Candidate A and B
      Compare the hand-count sampling totals for Candidate A and B with the actual
            totals for Candidate A and B to assess the accuracy of the hand-count
            sample
      If the difference between the hand-count sample and the actual, unaltered
            tally is within 1% of the vote, consider the hand-count test a success
      If the difference between the hand-count sample and the actual, unaltered
            tally is greater than 1% of the vote, consider the hand-count test a
            failure
```

One key finding is that there is no one hand-count sample percentage that is perfectly suited for accurately validating every size of election contest. While a random sampling of 0.5% of the ballots in each precinct in a ten million vote contest in a state like California is highly accurate, a sample of

[24] The optical scan ballot is highly preferable to a mere "paper trail" for a number of reasons: it is marked by the voter and does not depend on a separate step of voter verification; it is far easier to number and count than most paper trail formats such as thermal rolls; it is not dependent on printer functions highly subject to breakdown, etc.

5% of the ballots in every precinct is not sufficient to ensure 99% confidence of plus or minus 1% accuracy in a U.S. Congressional election of 200,000 voters.

As a rule of thumb, regardless of the size of the jurisdiction, sampling approximately 15,000 total ballots is sufficient to reach 99% confidence of ±1% accuracy.

Technical notes

The results reported here were generated by an election simulation program written in Microsoft Visual Basic.NET; that language was chosen simply for ease of rapid programming. The algorithms are described in sufficient detail to enable creation of a functionally-equivalent simulation in any alternative language of choice.

Many aspects of the simulation are governed by pseudorandom number generators. No programming language has a perfect random number generator, but the results of this simulation show very small deviations from perfect randomness.